APPENDIX

Angle Factors

When one considers first the angle factor F_{dA_x-1}, the general expression (9*a*) for surfaces having one infinitely elongated dimension can be applied to the parallel plate and adjoint plate systems of Figures 1(a) and 1(b), respectively, yielding

$$F_{dA_x-1} = \frac{1}{2}\left\{\frac{1-X}{\sqrt{(1-X)^2+\gamma^2}} + \frac{X}{\sqrt{X^2+\gamma^2}}\right\} \quad (A1)$$

$$F_{dA_x-1} = \frac{1}{2}\left\{1 + \frac{\cos\theta - X}{\sqrt{1-2X\cos\theta+X^2}}\right\} \quad (A2)$$

For the parallel disk system the derivation is carried out with angle factor algebra as in Equations (33) and (34) of reference 4, with the result

$$F_{dA_x-1} = \frac{1}{2}\left\{1 - \frac{\nu^2-1+X^2}{\sqrt{[\nu^2+1+X^2]^2-4X^2}}\right\} \quad (A3)$$

The angle factor F_{2-1} for interchange between two finite surfaces is obtained by direct integration in accordance with Equation (11*a*). The end results for the parallel plate, adjoint plate, and parallel disks system are respectively as follows:

$$F_{2-1} = \sqrt{1+\gamma^2} - \gamma \quad (A4)$$

$$F_{2-1} = 1 - \sin(\theta/2) \quad (A5)$$

$$F_{2-1} = \frac{1}{2}\{2+\nu^2 - \nu\sqrt{\nu^2+4}\} \quad (A6)$$

Next when one considers the infinitesimal angle factor $dF_{dA_y-dA_x}$, application of Equation (9*b*) to the parallel plate and adjoint plate systems yields

$$dF_{dA_y-dA_x} = \frac{\gamma^2}{2}\frac{dX}{[(X-Y)^2+\gamma^2]^{3/2}} \quad (A7)$$

$$dF_{dA_y-dA_x} = \frac{1}{2}\frac{XY(1-\cos^2\theta)\,dX}{[X^2+Y^2-2XY\cos\theta]^{3/2}} \quad (A8)$$

The corresponding result for the parallel disks is taken directly from Equation (36) of reference *4*:

$$dF_{dA_y-dA_x} = 2\nu^2\frac{(\nu^2+X^2+Y^2)\,X\,dX}{[(\nu^2+X^2+Y^2)^2-4X^2Y^2]^{3/2}} \quad (A9)$$

Finally the double integral appearing in the last term of Equation (17) may be carried out in closed form for the adjoint plate and parallel disk systems. The respective results of the integration are

$$\frac{1}{L}\int_0^L I\,dA = 1 - \sin(\theta/2) + \frac{1}{8}(\theta-\pi)\sin\theta \quad (A10)$$

$$\frac{1}{\pi R^2}\int_0^R I\,dA = \frac{1}{2}\left\{2+\nu^2 - \nu\sqrt{\nu^2+4} - \nu\tan^{-1}\frac{\nu}{2} + \nu\tan\left(\frac{\nu}{2}-\frac{1}{2\nu}\right)\right\} \quad (A11)$$

The corresponding integration for the parallel plate system could not be carried out in closed form.

Prediction of Pressure Drop in Two-Phase Single-Component Fluid Flow

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Data on pressure drop in two-phase, single-component fluid flow, both with and without heat transfer, are presented in terms of the Lockhart and Martinelli correlation parameters. The fluids used were trichloromonofluoromethane and hydrogen.

The results are compared with the correlation curve recommended by Martinelli and Nelson and give frictional pressure drops that are about 40% lower than the curve. The reasons for the deviations are discussed in terms of the effects of friction factors, the existence of metastable equilibrium, accuracy of data and instrumentation, and calculation procedures.

It is concluded that the Martinelli and Nelson correlation and a simple momentum pressure drop computation can be superposed to predict roughly the total pressure drop in tubes containing steady state, two-phase, single-component fluid flow with appreciable vaporization.

The fluid mechanical design of piping systems is handicapped by lack of information concerning so-called *steady state*, two-phase, single-component fluid flows. The objective of the investigation described here was to provide some of the required information. Data have been obtained and correlated for two-phase, single-component fluid flows, both with and without heat transfer. The results of some experiments with liquid hydrogen flowing with large heat flux (*11*), as well as those obtained from an extensive investigation with trichloromonofluoromethane, are reported. For the latter investigation the quality was varied from zero to unity; the Reynolds number based upon the total mass flow being saturated liquid (that is zero quality), was varied from about 5,000 to 90,000; the pressure and temperature were varied so that fluid properties changed significantly; and the diameter of the test section was varied. In addition single-phase (liquid) friction factors, which differ appreciably from published data (*10*), were determined.

Because the mathematical models required for theoretical analysis of two-phase flow problems are complex, the more successful approaches have been empirical and semiempirical. The majority of theoretical analyses, for example Harvey and Foust (*3*), have assumed that the fluid is homogeneous. The work of Linning (*7*) is an example of a semiempirical approach; Linning set up one-dimensional models for both stratified and annular flow configurations, determining unknown parameters experimentally. Lockhart and Martinelli (*8*) present a moderately successful correlation for the two-component problem.

Rogers (*12*) utilized the equations of Harvey and Foust to predict the behavior of two-phase hydrogen flows in well-insulated transfer systems; his results have not been experimentally verified. A large amount of computation is required to obtain numerical results with the equations of Harvey and Foust; Rogers used a high-speed computer.

Martinelli and Nelson (9) adapted the two-component correlation of Lockhart and Martinelli to single-component systems by a simple correction to account for changes in the axial component of momentum. The two-component systems of Lockhart and Martinelli do not involve appreciable mass transfer between the phases, whereas the single-component systems of Martinelli and Nelson and the present authors do. The potential advantage of this approach over that of Harvey and Foust is the relative simplicity of the computations. However Martinelli and Nelson did not possess sufficient experimental data to verify their approach.

In one of the more recent studies on the prediction of pressure drop in two-phase, single-component fluid flow Isbin et al. (4) conducted extensive experimental work with steam-water mixtures and investigated various means (including that of Martinelli and Nelson) for correlating their results.

EXPERIMENTAL INVESTIGATIONS

Description of Apparatus

The apparatus, shown schematically in Figure 1, is a closed-loop system that permits steady state runs of long duration. Trichloromonofluoromethane was chosen as the test fluid for two reasons. As its saturation temperatures at the operating pressures are essentially ambient, the required energy source and sink can be house-heating steam and cold water. Since the temperature difference between the fluid and the atmosphere is small, insulation for the adiabatic tests presents no problem. In addition when the mode of flow is to be observed through transparent test sections, essentially adiabatic tests can be conducted with no insulation.

The somewhat arbitrary use of trichloromonofluoromethane is justified by the assumption that the fundamental phenomena do not depend upon the fluid. If this assumption is valid, the bulk of the experimental work can be done with economical experiments of the type described here, only a few check points with other fluids being required.

The apparatus consists of a pump, a heating system, test sections, condensing systems, a receiver, and instrumentation.

A centrifugal water pump establishes and maintains flow through the apparatus. This flow is measured with a rotameter.

The pump discharges into a preheater and saturator which use low-pressure steam to heat and vaporize the fluid. The preheater permits better control of the saturator pressure. The saturator heats the liquid (as closely as possible) to its saturation temperature. The vapor formed in the saturator leaves through the vent return, is condensed and subcooled, measured with a rotameter, and returned to the receiver. The saturator pressure is regulated with the vent return valve. The temperature and pressure in the saturator, near the entrance to the test section, are measured with a dial thermometer and a mercury manometer. The flow from the saturator is guided into the test section by a well-rounded nozzle.

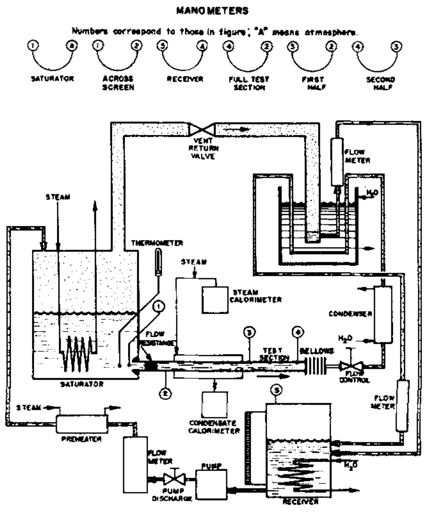

Fig. 1. Schematic of two-phase flow apparatus.

Flow resistances (screens and baffles) are installed between the nozzle and the test sections. These tend to produce closer approaches to states of stable equilibrium (6) and provide a means for varying the quality of the fluid in the test sections.

The test section consisted of upstream and downstream sections. For about three quarters of the runs the test section was essentially a singe piece of tubing, a pressure and temperature measuring station at the approximate middle separating the upstream from the downstream section.

The adiabatic pressure-drop data were taken in the downstream test section, insulated with 2 in. of fiber glass. The upstream section developed the flow for entrance into the downstream section. As the qualities attainable by flashing alone were limited to about 0.17, the upstream test section was heated with steam for about one quarter of the runs. Qualities to unity, and even superheated vapor, were then attainable. The data with heat transfer were obtained with this test section. The energy transferred to the fluid in this section was measured with two water-cooled calorimeters. The enthalpy of the entering steam was determined by the steam calorimeter, while the enthalpy of the leaving steam-water mixture was determined by the condensate calorimeter. Temperatures were measured with mercury thermometers, steam flows by weighing condensate, and cooling water flows with rotameters.

The heated test section was 0.438 in. I.D. with an equivalent hydraulic length of 31.5 ft. (actual length about 10 ft. with 6 U-bends). The adiabatic test sections were 0.552 in. I.D. × 2.53 ft. long, 0.522 in. I.D. × 3.50 ft. long, 0.438 in. I.D. × 3.42 ft. long, and 0.325 in. I.D. × 3.25 ft. long. The 0.522 in. I.D. section was glass; the others were copper.

Three differential manometers were used: across the nozzle and flow resistances, across the upstream test section, and across the downstream test section. Copper constantan thermocouples, mounted on the outside surface of the metallic test sections (at stations 2, 3, and 4 in Figure 1), measured the fluid temperature during the adiabatic tests.

A stainless steel bellows permitted the necessary thermal expansion. The flow through the test sections was controlled by a needle valve. After leaving the test sections the fluid was condensed and subcooled, metered by a rotameter, and returned to the receiver.

The receiver and saturator are fitted with sight glasses to measure fluid accumulation. The receiver contains a cooling coil which assists in preventing cavitation in the pump.

Experimental Procedure

Single-Phase Flow Experiments. Single-phase-liquid friction factors were measured because an accurate knowledge of them is required for determination of the effective length of the heat transfer test section and for application of the Martinelli and Nelson correlation

Steady single-phase isothermal flow was obtained by pressurizing the saturator with nitrogen gas. The trichloromonofluoromethane was cooled by circulating cooling water through the saturator coils, the upstream test section jacket, and all of the condensers and subcoolers (refer to Figure 1).

The following procedure was used for obtaining the data. The liquid level in the receiver was read, and a stop watch was started; the manometer, pressure gauge, flow meter, and thermocouple readings were recorded; the level in the receiver was reread, and the stop watch was stopped. Finally the saturator liquid temperature was read. Each experimental point is the arithmetic average of ten sets of readings. This scheme was followed for several flows until the range of flow was covered. These tests were conducted with the smallest and largest diameter test sections.

Two-Phase Flow Experiments. The flow of water through the water-cooled equipment was established, and the pump was started. When the liquid level in the saturator was several inches above the nozzle, steam was admitted to the preheater and the saturator. As the saturator pressure increased, the saturator level was kept relatively stationary with the pump discharge and the flow control valves. When the saturator pressure attained a preselected value, the vent return valve was adjusted to maintain this value, the steam to the preheater and saturator being adjusted if necessary. The steam flow to the first test section jacket was started. A preselected value of the test section flow rate was obtained with the flow control valve. The data were recorded after the attainment of steady state conditions.

The independent variables (flow rate, pressure level, quality, and test section diameter) were varied as follows. With

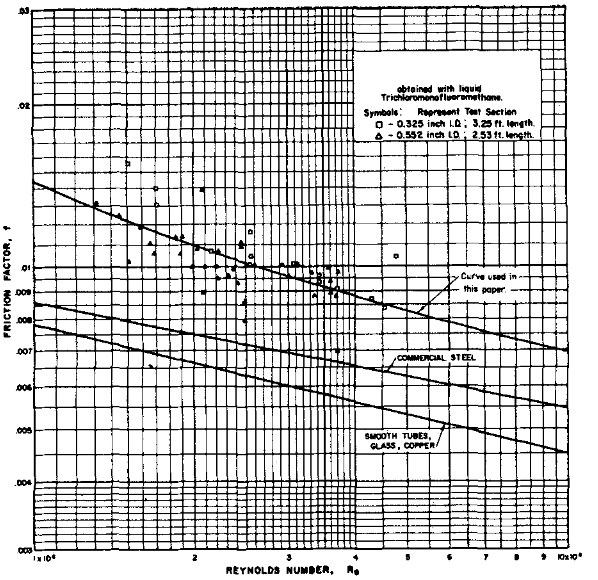

Fig. 2. Friction factor data.

all other independent variables held constant, the flow rate was varied through its range, a complete set of data (ten sets of readings) being recorded at each flow rate. Then the saturator pressure was changed; at each pressure level data were obtained for each flow rate. The quality was permitted to vary from run to run but was constant for each run. It was altered by using various resistances and by varying the steam flow to the heater on the upstream test section. The complete ranges of pressure level and flow rate were covered for each resistance and/or steam flow rate. Finally, after all values of these parameters were covered, a different diameter was used. All values of each variable were used with all values of all of the other variables.

The manometer, thermocouple, thermometer, and rotameter readings were recorded for each set of readings. The liquid accumulation in the receiver was obtained by timing the receiver level change that occurred while the other readings were being taken.

Accuracy of Measurements

The reading errors given in the following discussion are maxima. Because the data used for each run are the arithmetic average for ten sets of readings, the probable error for each run is less. The other errors, which are more difficult than the reading errors to evaluate, were minimized by the usual methods. For example fluid was forced through the test sections in both directions to detect the effects of poor pressure taps; thermocouples were checked at known temperatures; some of the rotameters were calibrated to check the manufacturer's accuracy claims; and the thermometers were in oil-filled wells that protruded well into the fluid whose temperature was measured. It is thought that the reading errors were the major source of inaccuracy.

All manometers could be read to 0.05 in. The reading error of saturator pressure was less than ½%. When the readings were very small, errors in the manometer readings across the test section could have been 20%; however when the flows were not small, these errors became negligible.

The reading errors associated with all flow meters were less than 5%; at the higher test section flows they were less than 1%. For some of the earlier runs the flow through the test section was computed by a difference method; the error in some of these computed flows could have been as high as 10%.

The saturator temperature could be read to within ½°F., introducing an error into our results of less than 1%. Thermocouple potentials were read to about 0.02 mv., which corresponds to about 0.90°F. The accuracy expected with the mercury thermometers, used with the calorimeters, is 0.05°F., introducing a maximum error of 1% in the quality computations.

CALCULATIONS

Three sets of calculations are used in this work. The first computes the liquid friction factor and the Reynolds number. The second determines the equivalent length of the heated test section. The third is used to compute the parameters for the Martinelli and Nelson correlation.

An IBM-650 data processor was used for many of the calculations because of the large volume of work involved.

Single-Phase Friction Factor

The friction factor and corresponding Reynolds number were calculated from data taken with liquid flowing through the downstream test section, the length of which was accurately known. The friction factor was computed from

$$f = \frac{\pi^2 D^5 \rho_{sc} \Delta p}{32 L (Q_r \rho_r)^2} \qquad (1)$$

The corresponding Reynolds number follows from

$$N_{Re} = \frac{4 Q_r \rho_r}{\pi D \mu'} \qquad (2)$$

Values of f, which are used in determining the equivalent length of the upstream test section and in the Martinelli and Nelson correlation, are plotted against N_{Re} in Figure 2.

Equivalent Length of Upstream Test Section

The equivalent length of the upstream test section was obtained by using single-phase flow pressure drop data collected between stations 2 and 3 (see Figure 1). The Reynolds number was calculated from Equation (2), and the friction factor was obtained from Figure 2. The equivalent length was obtained by solving Equation (1) for the length, L.

Two-Phase Flow Parameters

The Martinelli and Nelson parameters ϕ and $\sqrt{\chi}$ (each for turbulent

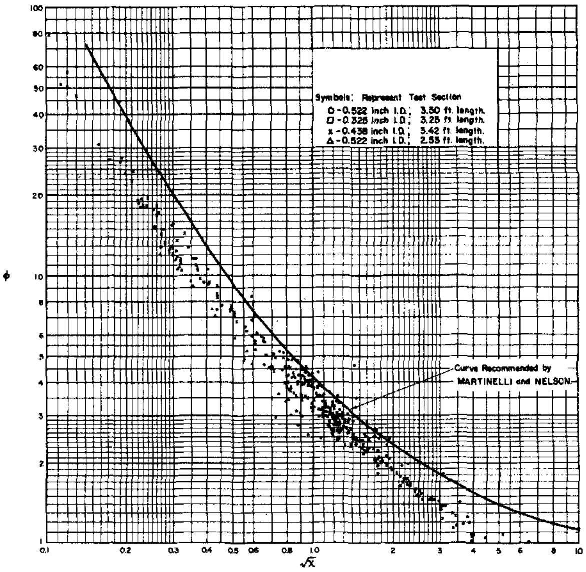

Fig. 3. Adiabatic tests with trichloromonofluoromethane.

flow of both phases) are calculated from the following equations. The detailed derivations of these relations are given in references 8 and 9.

The parameter ϕ is defined as

$$\phi = \left[\frac{\dfrac{\Delta p_f}{L}}{\left(\dfrac{\Delta p}{L}\right)_l} \right]^{\frac{1}{2}} \tag{3}$$

The method for calculating $(\Delta p_f)/(L)$ is deferred to Equation (11); the denominator of Equation (3) is computed from

$$\left(\frac{\Delta p}{L}\right)_l = \left(\frac{\Delta p}{L}\right)_o (1 - x_m)^{1.75} \tag{4}$$

where $\left(\dfrac{\Delta p}{L}\right)_o$ is given by the Fanning equation

$$\left(\frac{\Delta p}{L}\right)_o = \frac{4 f \rho_l u_o^2}{2D} \tag{5}$$

The authors' experimental data indicate that the friction factor and the Reynolds number are related by

$$f = \frac{0.2625}{(N_{Re})_o^{0.3204}} \tag{6}$$

The Reynolds number is calculated from

$$(N_{Re})_o = \frac{4\, Q_r \rho_r}{\pi D \mu_l} \tag{7}$$

The mean flow velocity is computed from

$$u_o = \frac{4\, Q_r \rho_r}{\pi D^2 \rho_l} \tag{8}$$

Combining the preceding equations one gets the final equation for ϕ:

$$\phi = \left[\frac{\pi^2 D^5 \rho_l \left(\dfrac{\Delta p_f}{L}\right)}{32 f\, (Q_r \rho_r)^2 (1 - x_m)^{1.75}} \right]^{\frac{1}{2}} \tag{9}$$

The friction pressure drop (Δp_f) is calculated by subtracting the pressure drop due to changes in the axial component of momentum (Δp_m) from the measured pressure drop (Δp_s):

$$\Delta p_f = \Delta p_s - \Delta p_m \tag{10}$$

The pressure drop due to momentum changes is computed by assuming that the liquid and vapor phases move with the same velocity. Thus

$$\frac{\Delta p_f}{L} = \frac{\Delta p_s - \dfrac{64 (Q_r \rho_r)^2}{\pi^2 D^4} \left(v_f^e - v_f^i + x^e v^e_{fg} - x^i v^i_{fg}\right)}{L} \tag{11}$$

As it is assumed that the phases are saturated fluids, values for v_f^e, v_f^i, v_{fg}^e, and v_{fg}^i are determined directly from the measured pressures and available thermodynamic data.

The pressures at the entrance and exit of the test sections are calculated from

$$p^i = \pi - \Sigma \Delta p \tag{12}$$

$$p^e = \pi - \Sigma \Delta p - \Delta p_s \tag{13}$$

The qualities at the various locations in the test section were calculated from energy balances:

$$x_2 = \frac{h_1 - (h_f)_2}{(h_{fg})_2} \tag{14}$$

$$x_3 = \frac{w_s[h_s - h_c] + w[h_1 - (h_f)_3]}{w (h_{fg})_3} \tag{15}$$

$$x_4 = \frac{w_s[h_s - h_c] + w[h_1 - (h_f)_4]}{w (h_{fg})_4} \tag{16}$$

The specific enthalpy of the liquid in the saturator (station 1) is

$$h_1 = h_{sat} - c_p (T_{sat} - T_1) \tag{17}$$

The values of the h_f's and h_{fg}'s are obtained from tables of properties, entering the tables at the measured pressures. The specific enthalpy of the steam is obtained by setting up an energy balance around the steam calorimeter:

$$h_s = (T_7 - T_0) + \frac{w_w}{w_t} (T_6 - T_5) \tag{18}$$

Similarly the specific enthalpy of the condensate is calculated by means of an energy balance around the condensate calorimeter:

$$h_c = (T_7 - T_0) + \frac{w_w}{w_s} (T_6 - T_5) \tag{19}$$

The mass flow rate through the test sections is given by

$$w = Q_r \rho_r \tag{20}$$

The cooling water flow rates were obtained with rotameters, and the steam and condensate flow rates were obtained by weighing.

It is assumed that the mean quality (x_m) is ½ $(x_2 + x_3)$ for the upstream test section and ½ $(x_3 + x_4)$ for the downstream test section. The consequences of this assumption are discussed later.

The parameter, $\sqrt{\chi}$, is calculated from

$$\sqrt{\chi} = \left(\frac{\rho_v}{\rho_l}\right)^{0.571} \left(\frac{\mu_l}{\mu_v}\right)^{0.143} \left(\frac{1}{x_m} - 1\right) \tag{21}$$

RESULTS

The principal results are the steady state, two-phase, single-component pressure drop data measured in these investigations; they are expressed in terms of the Martinelli and Nelson parameters and compared with the correlation of Martinelli and Nelson. The hydrogen data of Richards (*11*) are presented in the same manner. To assist in the discussion the friction factors and some data bearing on the existence of states of metastable equilibrium obtained in these investigations are presented.

The pressure drop results are presented in Figures 3, 4, and 5 on which the correlation parameters ϕ and $\sqrt{\chi}$ are plotted. The adiabatic data and the data with heat transfer for trichloromonofluoromethane are shown in Figures 3 and 4 respectively, while Richards' data for hydrogen are presented in Figure 5. The points represent the data, while the curve is the correlating curve recommended by Martinelli and Nelson. The trichloromonofluoromethane data were obtained with Reynolds numbers (based upon the total mass flow being saturated liquid) from 5,000 to 90,000, with qualities from zero to unity, and with saturation temperatures from 86° to 154°F. The heat transfer rates in the upstream test section were varied from zero to the rates required to vaporize all of the test fluid. [It should be noted that some results presented by the authors in a previous publication (5) differ from those presented here. The previous paper contained a computational error.]

The data of Richards were obtained with hydrogen flowing through uninsulated copper tubes; three inside diameters (0.118, 0.311, and 0.555 in.) and two test section lengths (44.9 and 130.4 in.) were used. The range of Reynolds numbers, based upon the total mass flow being saturated liquid, was from 18,000 to 600,000, while the range in average qualities was 0.014 to 0.39. The heat transfer rates varied from about 3,500 B.t.u./hr. sq.ft. to about 6,000 B.t.u./hr. sq.ft.

The friction factors are plotted against Reynolds numbers in Figure 2; for comparison the curves suggested by Perry (*10*) are also plotted. The friction factors for the authors' system are about 80% higher than those of Perry.

Figure 6 presents results which bear upon the deviation of two-phase flows from states of stable equilibrium. Data from two runs are given; for each run the measured fluid temperatures and the saturation temperatures corresponding to the measured static pressures are plotted. The deviation of

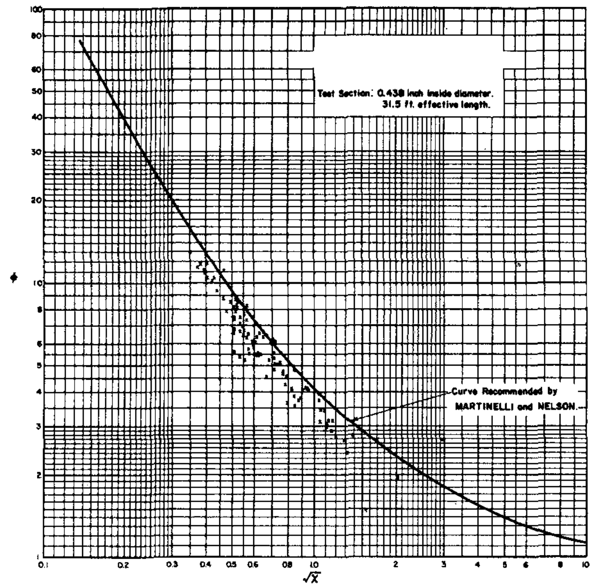

Fig. 4. Tests with heat transfer with trichloromonofluoromethane.

these curves from each other is a measure of the deviation from states of stable equilibrium.

DISCUSSION

The purpose of this discussion is to compare the results presented here with the Martinelli and Nelson correlation, to discuss the differences, and to evaluate the applicability of the correlation to steady state, two-phase, single-component, fluid flows with appreciable vaporization.

The effects of five factors will be discussed: the single-phase (liquid) friction factor, the existence of metastable equilibrium, the accuracy of the thermodynamic and transport properties data used, instrumentation, and calculation procedures.

For convenience the equations used to compute the Martinelli and Nelson correlation parameters are rewritten here. Combining Equations (9) and (11) one obtains

$$\phi = \left[\frac{\pi^2 D^5 \rho_l \left\{ \Delta p_s - \dfrac{64 (Q_r \rho_r)^2}{\pi^2 D^4} \left(v^e{}_f - v^i{}_f + x^e v^e{}_{fg} - x^i v^i{}_{fg} \right) \right\}}{32 f L \, (Q_r \rho_r)^2 \, (1 - x_m)^{1.75}} \right]^{\frac{1}{2}} \tag{22}$$

Rewriting Equation (21) one gets

$$\sqrt{\chi} = \left(\frac{\rho_v}{\rho_l} \right)^{0.571} \left(\frac{\mu_l}{\mu_v} \right)^{0.143} \left(\frac{1}{x_m} - 1 \right) \tag{21}$$

Perusal of Figures 3 and 4 shows that the data for both the adiabatic tests and the tests with heat transfer fall consistently below and to the left of the curve recommended by Martinelli and Nelson. For a given value of $\sqrt{\chi}$ it is seen that the deviation of the data from the curve varies from about 0 to 40%, the mean deviation being about 20%. Note that the corresponding mean deviation in pressure drops is about twice this amount. The shape of the recommended curve agrees with the shape of a curve which would represent the data.

Because the hydrogen data were obtained as a by-product from investigations that were primarily concerned with heat transfer, the pressure-drop instrumentation was crude; this can account for the scatter. However it can be seen that a mean curve through the points would fall below the curve recommended by Martinelli and Nelson. Thus the deviations observed for both the hydrogen and trichloromonofluoromethane data may be accounted for by the same explanations.

In the Martinelli and Nelson correlation there is an uncertainty of what friction factors should be used in the computations. The friction factor enters the correlation in two ways: the friction factor of the saturated liquid enters in Equation (22), and the exponents 1.75, 0.571, and 0.143 in Equations (22) and (21) are determined by the value of the exponent of the Reynolds number in the friction factor equation [Equation (6)].

Examination of the paper by Davidson (2), from which Martinelli and Nelson abstracted the data for their calculation, shows that the pure liquid friction factors measured by Davidson differ from Perry's curve in the same manner and to about the same extent as do the authors' measured friction factors. It seems probable therefore that the deviation of the authors' data from Martinelli and Nelson's curve is not due to the numbers used for the friction factor f in Equation (22). Spieth (*13*) obtained friction factors for liquid hydrogen flowing through well-insulated smooth pipes which agree with the curve which fits the data in Figure 2. There is therefore evidence that Perry's curve is not applicable to all systems. These variations in friction factors will not be discussed here; it should be noted however that Davidson's, Spieth's, and the authors' data were obtained with liquids close to saturation.

Another uncertainty arises from the value used for the exponent in the friction factor equations. Based upon Davidson's water data, Martinelli and Nelson used an exponent of 0.25, while the authors' data for pure liquid trichloromonofluoromethane were fitted by using an exponent of 0.3204. In the correlation (for turbulent–turbulent flow) it is assumed that the same friction factor equation applies to both liquid and vapor. In view of the facts that Davidson's and the authors' liquid friction data yielded different equations and that both of these differed from the generally accepted data in Perry, this assumption is questionable.

The authors use Martinelli and Nelson's exponents [refer to Equations (22) and (21)] in their calculations. Use of their exponent (0.3204) instead of Martinelli and Nelson's (0.25) changes the computed values of ϕ by from 0 to 3.5% and changes the computed values of $\sqrt{\chi}$ by from 4 to 5%. These changes would move the data points in Figures 3 and 4 down (away from) and to the right (towards) the Martinelli and Nelson curve. However it is apparent that errors introduced through the use of wrong values for the exponent in the friction factor equation would account for only a small part of the difference between the authors' results and those of Martinelli and Nelson.

The authors shall now consider the effects of the existence of states of metastable equilibrium and determine to what extent these effects can account for the deviations between their data and the curve recommended by Martinelli and Nelson. It is well known that when a vaporizing fluid flows through a tube, the drop in the local temperature can lag behind the drop in local static pressure resulting in a fluid which is superheated. This can be due to the appreciable time required for the formation of vapor and for the consequent reduction in temperature. Figure 6 shows that metastable states did exist in the authors' experiments. The data in the figure were obtained during adiabatic tests

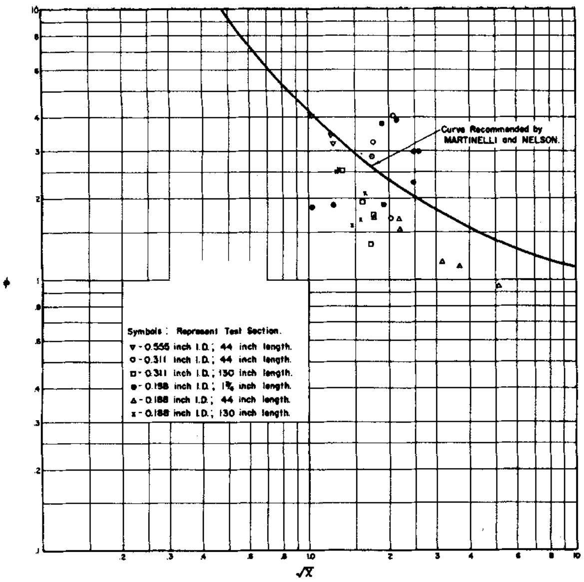

Fig. 5. Tests with heat transfer with hydrogen.

by means of thermocouples and manometers placed at 6-in. intervals along a thin-walled (0.010 in.) stainless steel test section. The thermocouples, placed on the outside of the tube wall under 2 in. of fiberglas insulation, gave temperatures accurate to better than 0.1°F. The manometers were connected differentially across each 6-in. interval and gave saturation temperatures accurate to about 0.1°F.

It should be recalled that the computations performed here depend upon the assumption that states of stable equilibrium exist at all times. The temperatures used in the computations were saturation temperatures corresponding to the measured pressures. Thus for those runs in which metastable states existed, the temperatures used in the computation were too low. The assumption is justified by the fact that in the design of piping systems for two-phase flows one is normally concerned with pressures, not temperatures; in fact if temperatures were to be used, one would be required to quantitatively predict metastable equilibrium states.

This use of temperatures which are too low affects the results, as presented in Figures 3 and 4, in two ways: the values of the densities (and specific volumes) and viscosities which are used in Equations (22) and (21) will be erroneous, and the values computed for the upstream, downstream, and mean qualities (x^i, x^e, and x_m respectively) in Equations (22) and (21) will be too large.

If both the actual temperature and the saturation temperature existing in the test section are known, then it is a simple matter, by using Equations (22) and (21) and thermodynamic data, to determine the error introduced by the assumption of stable equilibrium. The assumption produces values of ϕ which can be either too large or too small and values of $\sqrt{\chi}$ which are too small, the effect on $\sqrt{\chi}$ being the dominant one.

Calculations based upon a representative sample of the authors' data show that the error introduced into their computed values of ϕ, by the assumption of stable equilibrium, is in the range ±2%. The effects of f, which is dependent upon the values chosen for viscosity and density, and of x_m are small; the effects of the terms in the curley brackets in Equation (22) are dominant, the products being the more important. The sign of the error in ϕ depends upon the relative magnitudes of the errors in $x^e v_{fg}{}^e$ and $x^i{}_{fg}{}^i$; both signs resulted from the authors' data.

Calculations based upon the same representative sample of data show that the error introduced into the computed values of $\sqrt{\chi}$, by the assumption of stable equilibrium, was in the range of 0 to 12%. The assumption gives values of the density ratio [see Equation (21)] that are too small and values of the viscosity ratio that are too large; the error in the former can be as much as 3%, while that in the latter can be no greater than ¼%. Finally it is apparent that the error introduced into the mean quality (x_m) by the assumption of stable equilibrium serves to decrease the value of $\sqrt{\chi}$.

The foregoing discussion indicates that the errors introduced into the computation of the Martinelli and Nelson parameters by the assumption that the fluid is in stable equilibrium can account for as much as 1/3 of the deviation between the authors' results and the curves recommended by Martinelli and Nelson. The authors do not know if metastable states existed during the investigations upon which Martinelli and Nelson based their correlation. It is therefore impossible to assess the role of the assumption of stable equilibrium in the differences between the authors' results and those of Martinelli and Nelson.

The thermodynamic data of Benning and McHarness (*1*), used in processing the authors' data, are reported to be accurate to ±2%. However there is the possibility that the properties of the tricholoromonofluoromethane could have been altered by deterioration and contamination during the long period over which these investigations were conducted. It should be noted that because of leaks, modification and repairs, etc. it was necessary to periodically replenish the fluid in the apparatus. If a significant change took place in the fluid properties, it would have appeared as a time-dependent trend in the authors' results;

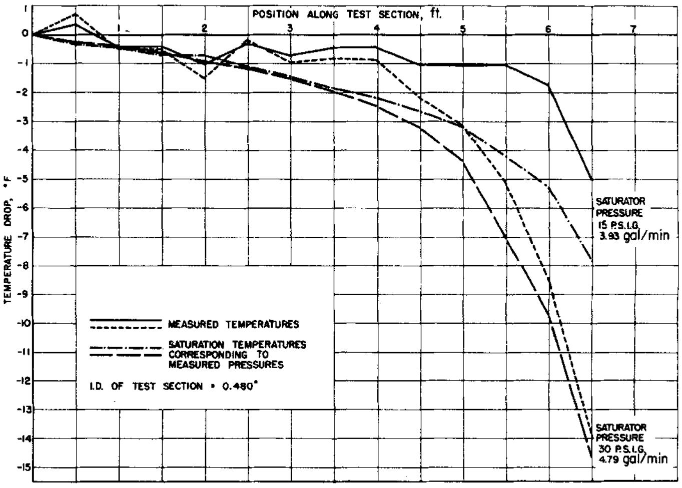

Fig. 6. Comparison of measured and saturation temperatures for trichloromonofluoromethane. Data shows the existence of metastable states.

this was not the case. In addition a spectrographic analysis of the fluid in the apparatus was not significantly different from that of a specimen of new fluid. Evaporation tests were conducted on 300-ml. samples of fluid from the apparatus and new fluid. The evaporation rates were the same, and only about 0.1 g. of greasy residue remained from the sample from the apparatus. The authors conclude that the values of the properties used in the computations were accurate to within $\pm 2\%$.

The errors involved in all of the instrumentation, except the thermocouples, affect the results presented in Figures 3 and 4. The accuracy of the instrumentation was discussed previously. Except for the very few runs in which the pressure differences and/or flow rates were extremely small, the errors involved in the measurements are too small to account for the deviation between the data plotted on the figures and the curve of Martinelli and Nelson.

The authors should examine the possibility that some of their calculation procedures might account for the differences between their results and those of Martinelli and Nelson. The errors in ϕ and $\sqrt{\chi}$ resulting from the instrumentation and property errors which are combined through the algebraic manipulations in Equations (22) and (21) are readily obtained and are not appreciable.

The assumption that both phases move with the same velocity (that is fluid is homogeneous) may introduce appreciable error into the computation of the momentum pressure drop (Δp_m); the computed values of Δp_m would be too large and the computed values of ϕ would be too small. This error could therefore account for the deviation between the authors' data and the Martinelli-Nelson curve. In the authors' experiments the momentum pressure drop varied from 0 to about 50% of the measured pressure drop. However the location and scatter of the data for relatively large momentum pressure drops were not distinguishable from the location and scatter of the data for relatively small momentum pressure drops. As a much greater error would probably be introduced when Δp_m is relatively large, it is concluded that the deviation between the authors' data and the Martinelli-Nelson curve is not caused by the homogeneity assumption. Two possible explanations for this result are: the tortuous flow path (6 U-bends in about 10 ft. of tube) in the test section where the large momentum pressure drops occurred prevented the phases from separating enough to achieve appreciably different velocities, and the boiling, which occurred in the tests where the momentum pressure drops were high, effectively mixed the phases.

The other sources of error arise in the computation of the average quantities used in Equations (22) and (21): the friction factor, the mean quality, the density ratio, and the viscosity ratio. Values of f, $\left(\frac{\rho_g}{\rho_l}\right)$, and $\left(\frac{\mu_l}{\mu_g}\right)$ are computed at a temperature which is the arithmetic mean of the saturation temperatures at each end of the test section, while x_m is the arithmetic mean of the qualities computed at each end of the test section. Because temperatures and pressures were measured at the ends of the test sections only, it is not possible to quantitatively evaluate the errors introduced by the arithmetic means. However from the shape of the curves in Figure 6 one can say that the arithmetic mean temperatures are probably too small and the arithmetic mean qualities are too high. As this would tend to increase the friction factor and decrease the quantity $(1 - x_m)^{1.75}$, the effect of the use of the arithmetic means upon ϕ is not apparent. The use of the arithmetic means in Equation (21) may make the density-ratio term too large, may not introduce significant error into the viscosity-ratio term, and may make the quality term $\left(\frac{1}{x_m} - 1\right)$ too small; thus the effect of the use of these means on $\sqrt{\chi}$ is also not apparent. To evaluate the errors that may be present in the authors' results it would be necessary to take measurements at more frequent intervals along the test section, as was done to obtain the data in Figure 6. This source of error was also present in the work upon which Martinelli and Nelson based their correlation; in fact the length-diameter ratios between their measurement stations were much greater than the authors. It should be noted that Martinelli and Nelson assumed that the quality varied linearly along the test section. Figure 6 indicates that this assumption may be very poor.

Essentially all of the remarks presented above are applicable to the hydrogen data of Richards (see Figure 5). The large scatter in the data is due to relatively crude instrumentation. In these experiments the momentum pressure drop (Δp_m) was as much as 90% of the measured pressure drop (Δp_s). It can be seen however that a mean curve through the points would be slightly below the curve recommended by Martinelli and Nelson. It is not known how much, if any, the hydrogen deviated from states of stable equilibrium. Also the kinetic energy terms, which may have been appreciable in the small pressure-drop runs, were omitted from the quality calculations.

Finally it should be pointed out that the water data of Isbin (4), when plotted on ϕ vs. $\sqrt{\chi}$ coordinates, agree closely with the recommended curve of Martinelli and Nelson. This leads to the possibility that the discrepancies between the authors' measurements and the Martinelli and Nelson curve may be due to the fact that the authors used a different fluid. However one should note that the range of the parameters during the authors' tests was sufficiently large to permit a significant variation of the properties of their test fluid and that there is no indication in their results that the points in Figures 3 and 4 are dependent upon the fluid properties appearing in Equations (21) and (22). It is therefore possible that the deviation between the authors' results and those of Martinelli and Nelson is due to other differences between the fluids and systems.

Examples of these differences are contamination, roughness, and nucleation and bubble growth characteristics which influence the existence of metastable equilibria; and the possibility that different modes of flow can exist with different fluids at the same point on the ϕ, $\sqrt{\chi}$ plane.

CONCLUSIONS

1. The Martinelli and Nelson correlation and a simple momentum pressure-drop computation can be superposed to predict roughly the total pressure drop in tubes containing steady state, two-phase, single-component fluid flow with appreciable vaporization. Mean curves through the new data (adiabatic trichloromonofluoromethane and both hydrogen and trichloromonofluoromethane with large heat fluxes) fall 10 to 30% below the curve (in the ϕ, $\sqrt{\chi}$ coordinate system) recommended by Martinelli and Nelson.
2. As the data obtained for the different diameters in both the hydrogen and trichloromonofluoromethane tests appear to be randomly distributed in the ϕ vs. $\sqrt{\chi}$ plots, the Martinelli and Nelson correlation apparently accounts for the effects of diameter adequately.
3. The temperature and pressure ranges used in the trichloromonofluoromethane experiments were large enough to permit significant variation of the thermodynamic and transport properties appearing in the correlation. As there is no apparent grouping of the data with respect to properties, it is concluded that the Martinelli and Nelson correlation adequately accounts for these variations in properties.

4. As the water data of Isbin, et al, agree closely with the recommended curve of Martinelli and Nelson, which was based upon the water data of Davidson, but the hydrogen and trichloromonofluoromethane data do not, it is concluded that there are some uncontrolled parameters which introduce inaccuracies to the Martinelli and Nelson correlation.

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NOTATION

c_p = isobaric specific heat of test liquid evaluated at arithmetic mean of actual temperature of test liquid in saturator and saturation temperature of test liquid corresponding to the saturator pressure
D = inside diameter of the test section
f = friction factor of the liquid in the test section
h = specific enthalpy
h_c = specific enthalpy of the condensate leaving the test section jacket
h_s = specific enthalpy of the steam entering the test section jacket
L = length of the test section
p = absolute pressure
p_s = saturator gauge pressure
Q_r = volumetric flow rate of subcooled test liquid in the test section rotameter
N_{Re} = Reynolds number defined by Equation (2)
T_0 = temperature at which specific enthalpy equals zero
T_5 = temperature of cooling water entering a calorimeter
T_6 = temperature of cooling water leaving a calorimeter
T_7 = temperature of the condensate leaving a calorimeter
v = specific volume
u_0 = mean flow velocity of test fluid in a test section if all of the test fluid were liquid at the arithmetic average of temperatures at the ends of the test section
w_w = mass flow rate of cooling water through a calorimeter
w = mass flow rate of test fluid through the test sections
w_s = mass flow rate of steam (or condensate water) through the test section jacket and the condensate calorimeter
w_t = mass flow rate of steam through the steam calorimeter
x = quality (mass fraction of vapor) of a two-phase fluid
x_m = arithmetic average of the qualities at the inlet and exit stations of a test section

Greek Letters

Δp = single-phase pressure drop between the ends of a test section
$\left(\frac{\Delta p}{L}\right)_l$ = friction pressure drop per unit length which would occur in a test section if only the liquid portion of the test fluid were flowing
$\left(\frac{\Delta p}{L}\right)_0$ = friction pressure drop per unit length of test section if all of the test fluid were liquid in the same state as the liquid portion of the two-phase test fluid
$(\Delta p_f)/(L)$ = two-phase flow friction pressure drop per unit length evaluated over a test section of length L
Δp_m = pressure drop between ends of a test section due to momentum changes of the flowing two-phase test fluid
Δp_s = measured pressure drop, of the flowing two-phase test fluid between ends of a test section
μ = viscosity
μ' = viscosity of the test liquid at the arithmetic average of the temperature at the inlet and exit to the test section
Π = absolute saturator pressure
π = 3.142
ρ = density
ρ_{sc} = density of the subcooled test liquid at the arithmetic mean of the temperatures at the ends of a test section
ρ_r = density of subcooled test liquid in the test section rotameter
$\Sigma\Delta p$ = sum of all pressure drops from inside the saturator to the entrance of a test section
ϕ = Martinelli and Nelson parameter defined by Equation (3)
χ = Martinelli and Nelson parameter defined by Equation (21)

Subscripts

1 = property of test liquid in saturator
2 = property of test fluid between flow resistances and the entrance to the upstream test section
3 = property of test fluid leaving the upstream test section and entering the downstream test section
4 = property of test fluid leaving the downstream test section
f = property of saturated liquid
fg = change in specific property during isobaric vaporization
g = property of saturated vapor
l = property of the saturated liquid phase of a two-phase fluid, evaluated at the arithmetic average of the saturation temperatures at the ends of a test section
sat = property of saturated test liquid corresponding to the saturator pressure
v = property of the saturated vapor phase of a two-phase fluid, evaluated at the arithmetic average of the saturation temperatures at the ends of a test section

Superscripts

e = property at exit of test section
i = property at entrance to test section

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